Dispersive anomalous diffusive transport in ratchets with long-range correlated spatial disorder

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The effects of quenched disorder with long-range spatial correlations on the transport properties of an overdamped periodic ratchet are investigated. The modified Fourier-filtering method is applied to generate the long-range correlated spatial disorder with statistical properties $\overline{\eta(x)\eta(x')} \sim |x'-x|^{-\gamma}$, where γ is the correlation exponent. Small amounts of this kind of quenched disorder are introduced in the equation of overdamped motion of a continuous time system, and the first two moments $C_1(t)=\langle x(t)\rangle$ and $C_2(t)=\langle (x(t)-\langle x(t)\rangle)^2\rangle$ are calculated. We show that the drift velocity is almost independent of γ. However, as a consequence of the long-range spatial correlations, the dispersive anomalous diffusve motion $[C_2(t)\sim t^H]$ appears in ratchets, with the difussion exponent H $(1<H<2)$ being dependent on γ. Moreover, we show that both the amount of quenched disorder and the correlation degree can enhance the anomalous diffusive transport.

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The study of ratchets has received much attention due to its role in describing the unidirectional transport of molecular motors [1], in modeling nanoscale friction [2,3], in surface smoothening [4], and so on [5,6]. In the previous studies, the drift velocity for describing the transport of the particles in asymmetric periodic potentials (ratchets) was studied by taking into account the time correlated fluctuations (noise) [7]. The transport properties were also investigated in deterministic, overdamped (or inertia) ratchets including a time-periodic force [8–10], and chaotic transport and current reversals could be found in these systems even in the absence of fluctuations (noise). More recently, quenched disorder was found to induce normal chaotic diffusive transport in overdamped and underdamped rachets [11,12]. However, in these studies, the force due to quenched disorder was introduced in the equation of motion through the presence of uniformly distributed random variables with no spatially correlations, and the effects of spatially correlated disorder have been neglected [11–14]. In fact, in many real disordered materials, such as polymers, porous materials, and amorphous systems [15], the spatial disorder is often correlated. For example, long-range spatial correlations have been found in a wide number of systems, including biological, physical, economical, geological, and urban systems [16,17]. In this paper, we report that the quenched disorder with long-range spatial correlations can induce a dispersive anomalous diffusive transport in periodic ratchets.

The motion of an overdamped particle, driven by an external periodic force on a ratchet with quenched disorder, is considered with the form

$$\eta\frac{dx}{dt}=-\frac{dU(x)}{dx}+\Gamma\sin(\omega t)+\alpha\xi(x), \tag{1}$$

where η, Γ, and ω are, respectively, the damping coefficient, the amplification, and frequency of the external force. $\alpha\xi(x)$ is the fluctuation force due to the quenched disorder with long-range spatial correlations, with α $(\alpha\geqslant 0)$ being the strength of disorder. In the present work, the variables $\xi(x)$ are assumed to be long-range, power-law spatial correlated, and are governed by the following relationship:

$$\overline{\xi(x)}=0 \quad \text{and} \quad \overline{\xi(x)\xi(x')}\sim|x'-x|^{-\gamma}, \tag{2}$$

where the correlation exponent γ is chosen to be in the range $0<\gamma<1$ in one-dimensional motional system [17]. Note that the quenched disorder term $\alpha\xi(x)$ remains constant with time on the period of the potential $[2k\pi, 2(k+1)\pi]$ with $k \in \mathbf{Z}$.

Without loss of generality, the unperturbed ratchet potential is chosen to be [8,11]

$$U(x)=-\sin(x)-\mu\sin(2x), \tag{3}$$

where μ is the asymmetry parameter with $0<\mu<1$.

As the force due to quenched disorder with long-range spatial correlations will modify the potential (3), it is natural to investigate how long-range spatial correlations affect the chaotic diffusion. In order to introduce the random force with long-range spatial correlations into the calculations, the modified Fourier-filtering method (MFFM) is adopted to generate the random variables $\xi(x)$ [16]. In what follows, we will normalize the long-range random numbers to have $\triangle\xi=\sqrt{\overline{\xi_i^2}-\overline{\xi_i}^2}=1$. To test the validity of MFFM, we calculate the spatial correlation function $\overline{\xi_i\xi_0}$ [i.e., $\overline{\xi(i)\xi(0)}$) averaging over different realizations of $L=2^{21}$ random numbers in Fig. 1. It is seen that when the spatial correlation is weak, i.e., γ is large, there is a good agreement between the numerical simulation data (solid symbols) and the theoretical curve $l^{-\gamma}$ (hollow symbols). As γ decrease (the spatial correlation degree becomes strong), the numerical simulations can still qualitatively characterize the theoretical curve well, although some discrepanices are found especially for the

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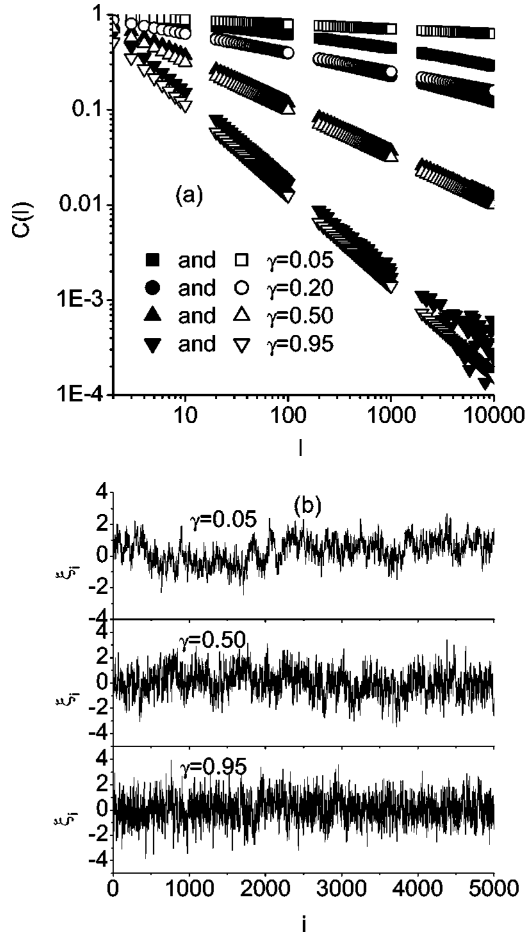

FIG. 1. (a) A log-log plot of the spatial correlation function $C(l)=\overline{\xi_l \xi_0}$ of 100 correlated samples with MFFM (solid symbols) and theoretical curve $l^{-\gamma}$ (hollow symbols) for $L=2^{21}$ and for different γ. (b) The random variables with long-range spatial correlation ξ_i as a function of i for different γ.

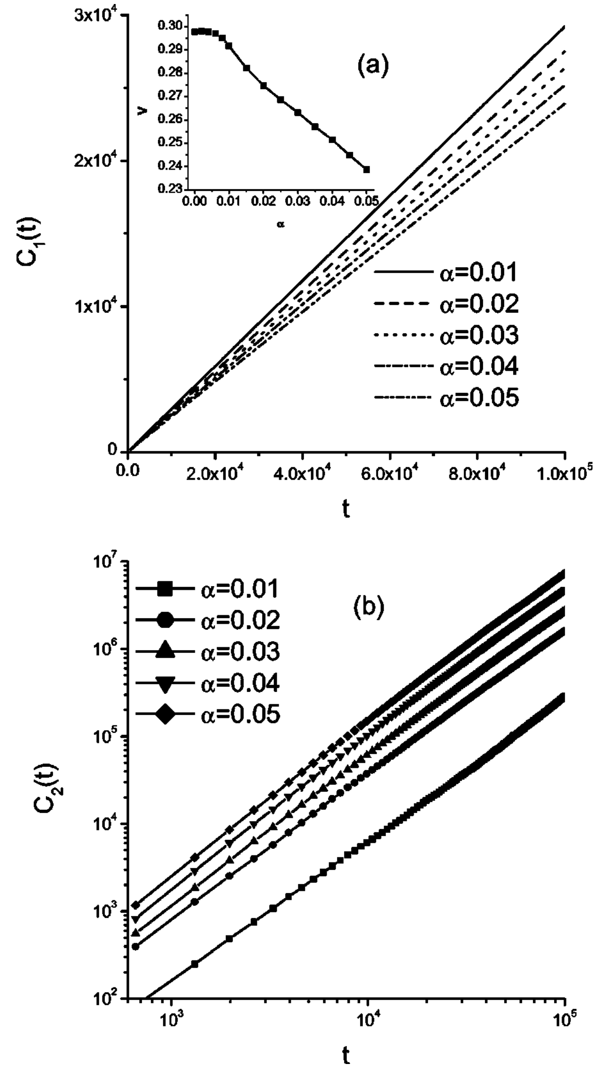

FIG. 2. The first and second moments $C_1(t)$ and $C_2(t)$ as a function of time t for $\Gamma=1.5$ and $\gamma=0.2$. In the inset, the drift velocity V is plotted as a function of α.

strong spatial correlation case at $\gamma=0.05$. In order to decrease such discrepancies, a more efficient filtering algorithm based on wavelets [18] can be adopted. In Fig. 1(b), it can be easily seen that smaller the correlation exponent γ, the more smooth the landscapes of the disorder sequences ξ_i become.

We choose $\eta=1.0$, $\mu=0.25$, and $\omega=0.1$, so that the system shows nonzero drift velocity if the force due to the quenched disorder is absent $(\alpha=0)$. When we include the quenched disorder force $\alpha\neq 0$, the solutions of Eq. (1) can show a complex behavior including the chaotic motion. Thus, averages of $\langle x(t)\rangle$ and $\langle (x(t))^2\rangle$ are performed over the ensemble of trajectories, which include different realizations of disorder and the spatial distribution of the positions of a particle. For numerical details, we use a variable step Runge-Kutta method [19] to integrate Eq. (1) and calculate the time series $x(t)$ for 5000 trajectories starting from different initial conditions centered around the origin $x=0$. Then averages such as $\langle x(t)\rangle, \langle (x(t))^2\rangle$ are performed over these ensembles. The ensemble described above is left to evolve for 4000 external by driven periods.

It has been shown that, in the case of quenched disorder force with no spatial correlations, the normal chaotic diffusive transport was observed in addition to the drift motion in the periodic ratchet [11,14], described by the linear dependence of both the first and second moments $C_1(t)=\langle x(t)\rangle$ and $C_2(t)=\langle (x(t)-\langle x(t)\rangle)^2\rangle$ on time t. Here, as a first step, we calculate $C_1(t)$ and $C_2(t)$ as functions of t for different values of disorder parameter α and for a fixed correlation exponent $\gamma=0.2$, and plot them in Fig. 2. It is found that, from Fig. 2(a), $C_1(t)$ is linear with time t, i.e., $C_1(t)\sim Vt$ with V being the drift velocity. Moreover, it can be seen that the value of $C_1(t)$ decreases with increasing the strength of disorder α, and there is no change in the direction of drift velocity for all values of α. The linear dependence of $C_1(t)$ on t allows us to investigate the drift velocity V as a function of α, as shown in the inset of Fig. 2(a). It is evident that the drift velocity decreases slightly when the disorder is in the range of $0<\alpha<0.01$, and then decreases significantly when $\alpha>0.01$. Thus, spatially correlated disorder can also suppress the drift velocity of a particle in a periodic ratchet driven by an external periodic force. It is quite interesting to find that, from Fig. 2(b), when the quenched disorder with

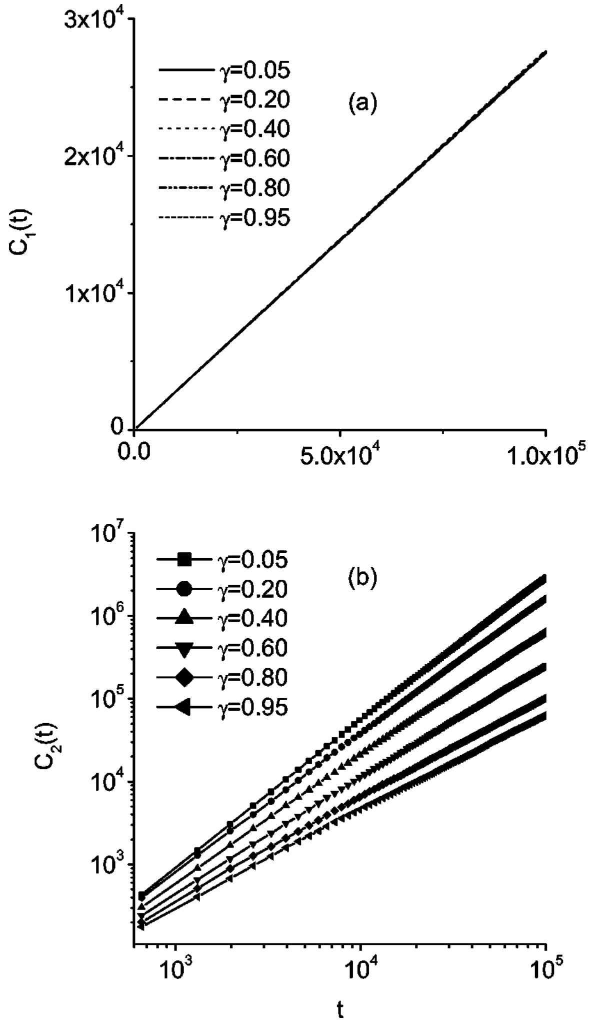

FIG. 3. $C_1(t)$ (a) and $C_2(t)$ (b) as a function of time t for $\Gamma=1.5$ and $\alpha=0.02$.

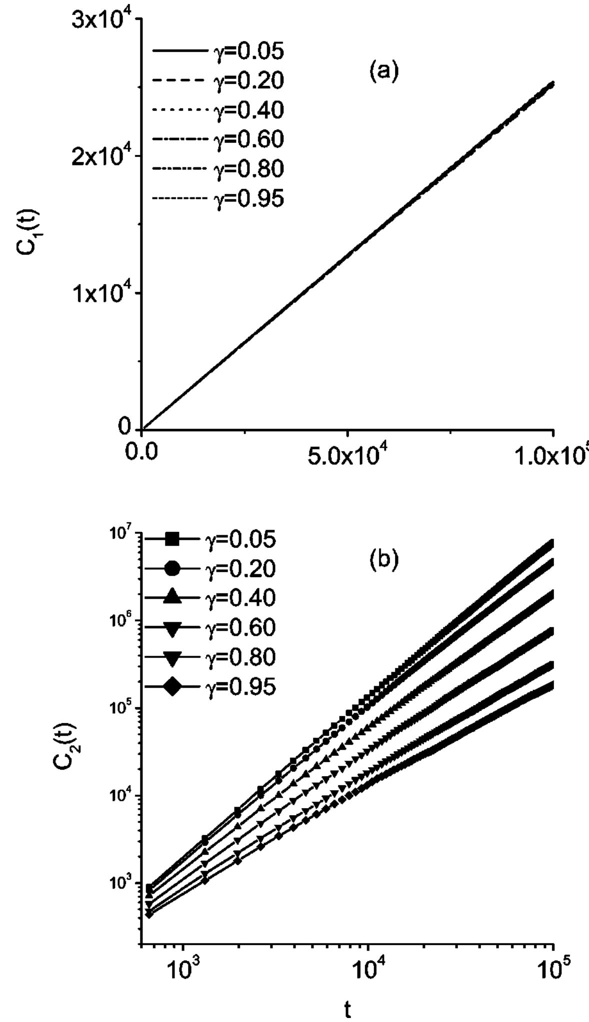

FIG. 4. Similar to Fig. 3, but for $\alpha=0.04$.

long-range spatial correlations is taken into account, the anomalous diffusive motion occurs due to the linear dependence of the natural logarithm of variance $C_2(t)$ on the natural logarithm of time t [$C_2(t)\sim t^H$, with the diffusion exponent $H>1$]. Furthermore, with the increase of α, the spatial fluctuations $C_2(t)$ increase significantly, in other words, the diffusionlike motion is greatly enhanced.

Thus we conclude that the strength of disorder can enhance the anomalous chaotic diffusion, but suppress the drift velocity. Such a conclusion is quite similar to the one without the spatial correlation [11]. This can be understood as follows. For small quenched disorder α, the drift term should not change significantly, even the correlated disorder will induce just small perturbations of the potential landscape.

In order to know the effects of the correlation degree γ on the transport of a particle, $C_1(t)$ and $C_2(t)$ are plotted for different γ in Fig. 3 for $\alpha=0.02$ and in Fig. 4 for $\alpha=0.04$. From Figs. 3(a) and 4(a), one can see that there is no variance in the first moment $C_1(t)$ with increasing correlation exponent γ, and this thus indicates that the drift moment is indeed independent of the spatial correlation degree. The anomalous diffusive motion appears again for all values of γ, and the curves of $C_2(t)$ decrease as γ increases [see Figs. 3(b) and 4(b)]. Thus, the degree of long-range spatial correlations enhances the anomalous diffusive motion, but it plays no role in the magnitude of the drift velocity. This is a nontrivial effect, as the drift velocity will remain finite, although the anomalous chaotic transport is enlarged.

It is known that when the anomalous diffusion motion occurs, the range of diffusion exponent H is larger than 1. For $1<H<2$, the diffusion belongs to the dispersive one, while for $H>2$, the diffusion is the enhanced one [20,21]. Thus it is of interest to investigate the effect of correlation exponent γ on the diffusion exponent H. In Fig. 5, the diffusion exponent H, as a function of the correlation exponent γ for different Γ, is plotted. We only choose $\alpha=0.02$ in our numerical calculations. In fact, the numerical results for other values of α (not shown here) indicate that H is only weakly dependent on α. From Fig. 5, it is evident that, as γ increases, i.e., the spatial correlation degree decreases, H goes down steadily in the range 1.1–1.8. This suggests that the anomalous diffusion induced by the quenched disorder with long-range spatial correlations belongs to the dispersive one. As far as the effect for Γ is concerned, the curve for H

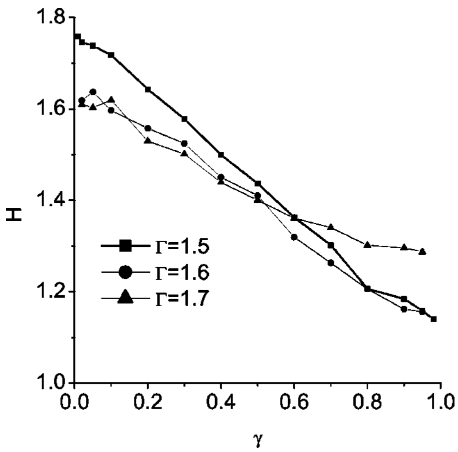

FIG. 5. The correlation exponent H as a function γ for $\alpha = 0.02$.

becomes flatter and flatter, and the values are found to be concentrated on small ranges as Γ increases (for example, $1.1 < H < 1.8$ for $\Gamma = 1.5$, while $1.3 < H < 1.65$ for $\Gamma = 1.7$). In fact, the dynamics of the particle is determined by the unperturbated ratchet potential, the potential due to the periodic force and the one due to the quenched disorder with long-range power-law correlations. With increasing Γ, the peroidic forces play a more important role in determining the motion of the particle and thus the influence of the long-range spatial correlations becomes relatively small, resulting in a weak dependence of H on γ.

In summary, we have shown that the long-range correlated spatial disorder on overdamped ratchets can lead to a dispersive anomalous diffusive motion in addition to a regular drift velocity. It is found that the increase of the strength of quenched disorder α can reduce the drift velocity, along with the enhancement of the anomalous diffusion. More importantly, the decrease of the correlation exponent γ (i.e., the increase of correlation degree) can further enhance the anomalous diffusion phenomenon, but plays no role in the drift transport. Our results show that the magnitude of the diffusion transport can be controlled by adjusting an alternative freedom, i.e., the correlation exponent γ. These conclusions may help us to understand the transport process or to interpret the experimental results of nanoscale frictions. However, in nanoscale frictions, the finite mass of the particles is important. Therefore, it would be of great interest to study the effects of quenched disorder with long-range spatial correlations on the dynamics of a ratchet with a finite mass.

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